

Preprint DFPD 97/TH/41
 hep-th/9709190
 September, 1997

SYMMETRY PROPERTIES OF SELF-DUAL FIELDS^a

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We briefly review the structure and properties of self-dual field actions.

The Lagrangian description of gauge fields which possess duality properties have been under study during a long period of time, a classical example being the electric-magnetic duality symmetry of the free Maxwell equations.

Present interest in this problem is caused by an essential progress in understanding an important role of dualities in quantum string theory and its effective field-theoretical limits. Self-dual and duality-symmetric fields are part of the spectrum in these theories and the knowledge of their effective actions would enable one to get more detailed information about the dynamical properties, symmetries and geometrical structure of the theory. And here a problem of how to construct a duality symmetric action arises. On a way of solving this problem one observes that self-dual (or duality-symmetric) fields have unusual group-theoretical properties and non-trivial topological structure inherent to this important subclass of gauge fields.

I shall describe this properties with the example of a self-dual antisymmetric 2-rank field $A_{mn}(x)$ in 6-dimensional space-time ($m, n = 0, \dots, 5$), though an approach¹ to be used is applicable to all known models with duality-symmetric fields in any space-time dimension of Lorentz signature.

A field $A_{mn}(x)$ is self-dual if

$$\partial_{[l} A_{mn]}(x) \equiv F_{lmn} = \frac{1}{6} \epsilon_{lmnpqr} F^{pqr} \equiv F_{lmn}^*. \quad (1)$$

This additional condition on the field strength of A_{mn} reduces twice the number of physical degrees of freedom of the gauge field which otherwise would have 6 transversal degrees of freedom. Upon imposing (1) A_{mn} has only three independent degrees of freedom. It is desirable to describe the dynamics of

^aTalk given at the Fifth International Wigner Symposium (Vienna, August 25-29, 1997)

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A_{mn} by use of the action principle, and consider (1) as the equation of motion of A_{mn} derived from an action. Since (1) is a first order differential equation one might admit that the action should be of the first order in derivatives. This is not typical of the bosonic fields whose free actions are usually quadratic in field derivatives (in a second-order formalism). If anyway we try to construct a first-order action, we shall realize that an infinite number of auxiliary fields are required² for the action to consistently describe a single self-dual field.

Another possibility is looking for a quadratic action constructed solely from components of F_{lmn} . Then, since equations of motion one derives from a quadratic action are of the second order in derivatives, one may expect that there should be additional local symmetry which would allow one to reduce these equations to the self-duality condition. Such actions have indeed been found for various models^{4,5} but they turn out to be nonmanifestly space-time invariant. In our case the self-dual action can be written as

$$S = \int d^6x \left[-\frac{1}{6} F_{lmn} F^{lmn} - \frac{1}{2} (F - F^*)_{0ab} (F - F^*)_0^{ab} \right], \quad (a, b = 1, \dots, 5) \quad (2)$$

It contains the ordinary kinetic term for A_{mn} , and the second term which breaks manifest Lorentz invariance down to its spatial subgroup $SO(5)$, since only time components of $(F - F^*)$ enter the action. However, it turns out that Eq. (2) is non-manifestly invariant under modified space-time transformations⁵ which (in the gauge $A_{0i} = 0$) look like

$$\delta A_{ab} = x^0 v^c \partial_c A_{ab} + x^c v^c \partial_0 A_{ab} - x^c v^c (F - F^*)_{0ab}. \quad (3)$$

The first two terms in (3) are standard Lorentz boosts with a velocity v_a , and the last term is a nonconventional one, it vanishes when (1) is satisfied, so that the transformations (3) reduce to the Lorentz boosts on the mass shell.

From (2) we get A_{mn} field equations, which have the form of Bianchi identities $\epsilon^{abcde} \partial_a (F - F^*)_{bc0} = 0$. Their general (topologically trivial) solution is

$$(F - F^*)_{ab0} = \partial_{[a} \varphi_{b]}(x). \quad (4)$$

If the right hand side of (4) was zero then $F_{ab0} = F^*_{ab0}$ and, hence, as one can easily check, the full covariant self-duality condition would be satisfied. And this is what we would like to get. One could put the r.h.s. of (4) to zero if there would be an additional local symmetry of (2) for which $\partial_{[a} \varphi_{b]} = 0$ is a gauge fixing condition. And there is indeed such a symmetry under

$$\delta A_{0a} = \hat{\varphi}_a(x), \quad \delta A_{ab} = 0, \quad \delta (F - F^*)_{ab0} = \partial_{[a} \hat{\varphi}_{b]}. \quad (5)$$

The existence of this symmetry is the reason why the quadratic action describes the dynamics of the self-dual field A_{mn} with twice less physical degrees of freedom than that of a non-self-dual one.

Thus we have constructed a non-manifestly Lorentz invariant action for the self-dual fields. But it is always desirable to have a covariant formulation, especially when one intends to consider more complicated models coupled to (super)gravity. This requires the use of auxiliary fields. Depending on the approach chosen their number vary from one¹ to³ infinity².

I will briefly describe a formulation with a single scalar auxiliary field $a(x)$ which has been used to construct the effective action for the M-theory 5-brane⁶. The covariant action for the $D = 6$ self-dual field looks as follows

$$S = \int d^6x \left[-\frac{1}{6} F_{lmn} F^{lmn} + \frac{1}{2(\partial_q a \partial^q a)} \partial^m a(x) (F - F^*)_{mnl} (F - F^*)^{nlr} \partial_r a(x) \right]. \quad (6)$$

In addition to ordinary gauge symmetry of $A_{mn}(x)$ the covariant action (6) is invariant under the following local transformations:

$$\delta A_{mn} = \partial_{[m} a \hat{\varphi}_{n]}(x), \quad \delta a = 0; \quad (7)$$

$$\delta a = \varphi(x), \quad \delta A_{mn} = \frac{\varphi(x)}{(\partial a)^2} \mathcal{F}_{mnp} \partial a^p. \quad (8)$$

The transformations (7) are a covariant counterpart of (5) and play the same role as the latter in deriving the self-duality condition (1).

Local symmetry (8) ensures the auxiliary nature of the field $a(x)$ required for keeping space-time covariance of the action manifest¹. An admissible gauge fixing condition $\partial_m a(x) = \delta_m^0$ for this symmetry reduces (6) to (2), the modified space-time transformations (3), which preserve this gauge, arising as a combination of the Lorentz boost and the transformation (8) with $\varphi = -v^c x^c$.

We have thus seen that the self-dual gauge fields possess more local symmetries than the ordinary gauge fields, and a nonpolynomial form of the covariant action (6), which is singular at $(\partial a)^2 = 0$, points to a nontrivial topological structure associated with these fields. Better understanding the nature of these symmetries and of the topological structure may be useful for studying quantum self-dual field theory.

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